1 Find all values of x in the interval $0 \le x \le 360°$ such that

a $\sin x = \frac{1}{2}$ **b** $\tan x = \sqrt{3}$ **c** $\cos x = 0$ **d** $\sin x = -1$

e $\cos x = \frac{\sqrt{3}}{2}$ **f** $\sin x = \frac{1}{\sqrt{2}}$ **g** $\tan x = -1$ **h** $\cos x = -\frac{1}{2}$

i $\sin x = -\frac{\sqrt{3}}{2}$ **j** $\tan x = \frac{1}{\sqrt{3}}$ **k** $\cos x = -\frac{1}{\sqrt{2}}$ **l** $\tan x = -\sqrt{3}$

2 Solve each equation for θ in the interval $0 \le \theta \le 360°$ giving your answers to 1 decimal place.

a $\cos \theta = 0.4$ **b** $\sin \theta = 0.27$ **c** $\tan \theta = 1.6$ **d** $\sin \theta = 0.813$

e $\tan \theta = 0.1$ **f** $\cos \theta = 0.185$ **g** $\sin \theta = -0.6$ **h** $\tan \theta = -0.7$

i $\cos \theta = -0.39$ **j** $\tan \theta = -3.4$ **k** $\cos \theta = -0.636$ **l** $\sin \theta = -0.203$

3 Solve each equation for x in the interval $0 \le x \le 360$.
Give your answers to 1 decimal place where appropriate.

a $\sin (x - 60)° = 0.5$ **b** $\tan (x + 30)° = 1$ **c** $\cos (x - 45)° = 0.2$

d $\tan (x + 30)° = 0.78$ **e** $\cos (x + 45)° = -0.5$ **f** $\sin (x - 60)° = -0.89$

g $\cos (x + 45)° = 0.9$ **h** $\sin (x + 30)° = 0.14$ **i** $\cos (x - 60)° = 0.6$

j $\sin (x - 30)° = -0.3$ **k** $\tan (x - 60)° = -1.26$ **l** $\sin 2x° = 0.5$

m $\cos 2x° = 0.64$ **n** $\sin 2x° = -0.18$ **o** $\tan 2x° = -2.74$

p $\sin \frac{1}{2}x° = 0.703$ **q** $\tan 3x° = 0.591$ **r** $\cos 2x° = -0.415$

4 Solve each equation for x in the interval $0 \le x \le 2\pi$ giving your answers in terms of π.

a $\sin x = 0$ **b** $\cos x = \frac{1}{2}$ **c** $\tan x = 1$

d $\cos x = -1$ **e** $\tan x = -\frac{1}{\sqrt{3}}$ **f** $\sin x = -\frac{1}{\sqrt{2}}$

g $\tan (x + \frac{\pi}{6}) = \sqrt{3}$ **h** $\sin (x - \frac{\pi}{4}) = \frac{1}{2}$ **i** $\cos (x + \frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$

j $\sin (x + \frac{\pi}{3}) = \frac{1}{\sqrt{2}}$ **k** $\cos 2x = -\frac{1}{\sqrt{2}}$ **l** $\tan 3x = \frac{1}{\sqrt{3}}$

5 Solve each equation for θ in the interval $-180° \le \theta \le 180°$.
Give your answers to 1 decimal place where appropriate.

a $\cos \theta = 0$ **b** $\tan 2\theta + 1 = 0$ **c** $\sin (\theta + 60°) = 0.291$

d $2 \tan (\theta - 15°) = 3.7$ **e** $\sin 2\theta - 0.3 = 0$ **f** $4 \cos 3\theta = 2$

g $1 + \sin (\theta + 110°) = 0$ **h** $5 \cos (\theta - 27°) = 3$ **i** $7 - 3 \tan \theta = 0$

j $3 + 8 \cos 2\theta = 0$ **k** $2 + 6 \tan (\theta + 92°) = 0$ **l** $1 - 4 \sin \frac{1}{3}\theta = 0$

6 Solve each equation for x in the interval $0 \le x \le 180°$.

Give your answers to 1 decimal place where appropriate.

a $\tan(2x + 30°) = 1$ **b** $\sin(2x - 15°) = 0$ **c** $\cos(2x + 70°) = 0.5$

d $\sin(2x + 210°) = 0.26$ **e** $\cos(2x - 38°) = -0.64$ **f** $\tan(2x - 56°) = -0.32$

g $\cos(3x - 24°) = 0.733$ **h** $\tan(3x + 60°) = -1.9$ **i** $\sin(\frac{1}{2}x + 18°) = 0.572$

7 Solve each equation for x in the interval $0 \le x \le 2\pi$, giving your answers to 2 decimal places.

a $\tan x = 0.52$ **b** $\cos 2x = 0.315$ **c** $\sin(x + \frac{\pi}{4}) = 0.7$

d $3\cos x + 1 = 0$ **e** $\sin \frac{1}{2}x = 0.09$ **f** $\tan 2x = -0.225$

g $3 - 4\sin(x - \frac{\pi}{3}) = 0$ **h** $\tan(2x + \frac{\pi}{6}) = 2$ **i** $\cos 3x = -0.81$

j $5 + 3\tan x = 0$ **k** $\cos(2x - \frac{\pi}{2}) = -0.34$ **l** $1 + 6\sin 2x = 0$

8 **a** Solve the equation

$$2y^2 - 3y + 1 = 0.$$

b Hence, find the values of x in the interval $0 \le x \le 360°$ for which

$$2\sin^2 x - 3\sin x + 1 = 0.$$

9 Solve each equation for θ in the interval $0 \le \theta \le 360$.
Give your answers to 1 decimal place where appropriate.

a $\sin^2 \theta° = 0.75$ **b** $1 - \tan^2 \theta° = 0$

c $2\cos^2 \theta° + \cos \theta° = 0$ **d** $\sin \theta°(4\cos \theta° - 1) = 0$

e $4\sin \theta° = \sin \theta° \tan \theta°$ **f** $(2\cos \theta° - 1)(\cos \theta° + 1) = 0$

g $\tan^2 \theta° - 3\tan \theta° + 2 = 0$ **h** $3\sin^2 \theta° - 7\sin \theta° + 2 = 0$

i $\tan^2 \theta° - \tan \theta° = 6$ **j** $6\cos^2 \theta° - \cos \theta° - 2 = 0$

k $4\sin^2 \theta° + 3 = 8\sin \theta°$ **l** $\cos^2 \theta° + 2\cos \theta° - 1 = 0$

m $\tan^2 \theta° + 3\tan \theta° - 1 = 0$ **n** $3\sin^2 \theta° + \sin \theta° = 1$

10 **a** Sketch the curve $y = \cos x°$ for x in the interval $0 \le x \le 360$.

b Sketch on the same diagram the curve $y = \cos(x + 90)°$ for x in the interval $0 \le x \le 360$.

c Using your diagram, find all values of x in the interval $0 \le x \le 360$ for which

$$\cos x° = \cos(x + 90)°.$$

11 **a** Sketch the curves $y = \cos x°$ and $y = \cos 3x°$ on the same set of axes for x in the interval $0 \le x \le 360$.

b Solve, for x in the interval $0 \le x \le 360$, the equation

$$\cos x° = \cos 3x°.$$

c Hence solve, for x in the interval $0 \le x \le 180$, the equation

$$\cos 2x° = \cos 6x°.$$

1 **a** Given that $4 \sin x + \cos x = 0$, show that $\tan x = -\frac{1}{4}$.

b Hence, find the values of x in the interval $0 \leq x \leq 360°$ for which

$$4 \sin x + \cos x = 0,$$

giving your answers to 1 decimal place.

2 **a** Show that

$$5 \sin^2 x + 5 \sin x + 4 \cos^2 x \equiv \sin^2 x + 5 \sin x + 4.$$

b Hence, find the values of x in the interval $0 \leq x \leq 360°$ for which

$$5 \sin^2 x + 5 \sin x + 4 \cos^2 x = 0$$

3 Solve each equation for x in the interval $0 \leq x \leq 360°$.
Give your answers to 1 decimal place where appropriate.

a $2 \sin x - \cos x = 0$

b $3 \sin x = 4 \cos x$

c $\cos^2 x + 3 \sin x - 3 = 0$

d $3 \cos^2 x - \sin^2 x = 2$

e $2 \sin^2 x + 3 \cos x = 3$

f $3 \cos^2 x = 5(1 - \sin x)$

g $3 \sin x \tan x = 8$

h $\cos x = 3 \tan x$

i $3 \sin^2 x - 5 \cos x + 2 \cos^2 x = 0$

j $2 \sin^2 x + 7 \sin x - 2 \cos^2 x = 0$

k $3 \sin x - 2 \tan x = 0$

l $\sin^2 x - 9 \cos x - \cos^2 x = 5$

4 Solve each equation for θ in the interval $-\pi \leq \theta \leq \pi$ giving your answers in terms of π.

a $4 \cos^2 \theta = 1$

b $4 \sin^2 \theta + 4 \sin \theta + 1 = 0$

c $\cos^2 \theta + 2 \cos \theta - 3 = 0$

d $3 \sin^2 \theta - \cos^2 \theta = 0$

e $4 \sin^2 \theta - 5 \sin \theta + 2 \cos^2 \theta = 0$

f $\sin^2 \theta - 3 \cos \theta - \cos^2 \theta = 2$

5 Prove that

a $(\sin x + \cos x)^2 \equiv 1 + 2 \sin x \cos x$

b $\frac{1}{\cos x} - \cos x \equiv \sin x \tan x, \quad \cos x \neq 0$

c $\frac{\cos^2 x}{1 - \sin x} \equiv 1 + \sin x, \quad \sin x \neq 1$

d $\frac{1 + \sin x}{\cos x} \equiv \frac{\cos x}{1 - \sin x}, \quad \cos x \neq 0$

6 **a** Prove the identity

$$(\cos x - \tan x)^2 + (\sin x + 1)^2 \equiv 2 + \tan^2 x.$$

b Hence find, in terms of π, the values of x in the interval $0 \leq x \leq 2\pi$ such that

$$(\cos x - \tan x)^2 + (\sin x + 1)^2 = 3.$$

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$$f(x) \equiv \cos^2 x + 2 \sin x, \quad 0 \leq x \leq 2\pi.$$

a Prove that $f(x)$ can be expressed in the form

$$f(x) = 2 - (\sin x - 1)^2.$$

b Hence deduce the maximum value of $f(x)$ and the value of x for which this occurs.

Solve for $\sin^2 Q + 3\sin Q = -2$ **over the interval** $0 \leq Q \leq 2\pi$

Possible Answers:

Q = 3π or does not exist 2

Q = π or 2π

Q = π or does not exist 2

Q = π or 3π2 2

Correct answer:

Q = 3π or does not exist 2

Explanation:

Substitute x = sinQ and solve the new equation $x^2 + 3x = -2$ by factoring. Be sure to change variables back to Q. As a result, sinQ = −1 or sinQ = −2. This function is bounded between −1 and 1 so sinQ can never be −2 and sinQ is −1 only at 3π/2 or 270 °.

If $sin(2x) = \frac{-\sqrt{2}}{2}$, **what is the value of** x **if** $\frac{\pi}{2} \leq 2x \leq \frac{3\pi}{2}$**?**

Possible Answers:

$\frac{2\pi}{3}$

$\frac{\pi}{4}$

$\frac{5\pi}{8}$

π

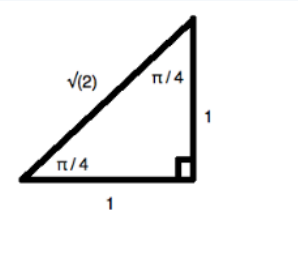
$\frac{5\pi}{4}$

Correct answer:

$\frac{5\pi}{8}$

Explanation:

Recall that the $45-45-90$ triangle appears as follows in radians:



Now, the sine of $\frac{\pi}{4}$ is $\frac{1}{\sqrt{2}}$. However, if you rationalize the denominator, you get:

$$\frac{1}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Since $sin(2x) = \frac{-\sqrt{2}}{2}$, we know that $2x$ must be represent an angle in the third quadrant (where the sine function is negative). Adding its reference angle to π, we get: